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THE MINIMIZATION OF AN AGRICULTURAL PRODUCT COST FUNCTION IN FERTILIZER EXPERIMENTS

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INTRODUCTION

Assuming that the productivity of an agricultural crop can be analyzed through a mathematical model (as a function of two or more nutrient levels, for example), researchers usually apply the Response Surface Methodology (RSM) (MYERS & MONTGOMERY, 1995) as a tool for estimation and prediction.

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When used as an optimization instrument, the chosen function often is a second order polynomial. It seems, at least for the authors of this work, that there is an implicit consensus between researchers and agricultural firms with regard to the optimum investment in fertilizers for a certain crop: "optimum" doses of fertilizer are those that maximize money profit in a broad sense (usually net profit or income rate over the amount invested in fertilizers). But mowadays, under the effects of day by day more "globalized" economies, it seems that the optimization of cost becomes an alternative strategy for "surviving" in such an environment. With this aim this work was prepared. It continues and expands an earlier work by PIMENTEL-GOMES & GARCIA (1995) in which they analyze the same problem with one explanatory variable (a mixture of nutrients).

MATERIAL AND METHODS

A function G(N, P, K) is defined. From now on, N, P and K refer to Nitrogen, Phosphorus (as P_2O_5) and Potassium (as K_2O), respectively. It is obtained by a ratio between a second order polynomial equation (the response surface fitted to productivity, based on experimental data) and a linear function of fixed and nutrient related costs. Symbolically, let Y(N, P, K) be:

$$Y = a_{000} + a_{001}N + a_{010}P + a_{100}K + a_{002}N^2 + a_{020}P^2 + a_{200}K^2 + a_{011}NP +$$
$$+a_{101}NK + a_{110}PK,$$

that is, the polynomial equation fitted to the productivity of a certain crop, and let D(N, P, K) be:

$$D = m + t_1N + t_2P + t_3K,$$

the linear function for the total costs (**m** denotes the fixed costs and $\mathbf{t_i}$, i = 1,2,3, the nutrient prices). According G(N, P, K) =Y/D, with nonnegative values for N, P and K.

If Y is measured in t/ha and D in R$/ha, then G yields the amount of product obtained for each real invested (although throughout this work the Brazilian currency, the real, was used, this methodology may be applied with any type of currency). It is easily seen that either the maximization of G(N, P, K) or the minimization of G^{-1} (N, P, K) yields the optimum levels of fertilizers that minimize the per unit product cost function.

Canonical analysis of G(N, P, K) is not a simple task for the coordinates of the critical point are given by a system of three second degree equations with three variables. An approximate solution can be obtained through a new fitting of a second order polynomial to a grid of points obtained from the G function. An interative process (new fittings) on the optimum response neighbourhood allows the point estimation of the optimum point to be as accurate as necessary.

One aspect that deserves careful attention is the estimation of the response surface for the productivity of the crop under analysis. Several authors warn about problems arising from the use of response surfaces in fertilizer-response experiments (see, for example, PIMENTEL-GOMES & CONAGIN, 1987; ZAGATTO & PIMENTEL-GOMES, 1960, 1967 and HEADY & DILLON, 1961), among others.

DATA SET

Data were taken from a previous work by MALAVOLTA **et al.** (1993) in which a multilocation trial was carried out to study foliar diagnosis in sugar cane. Fifteen 3^3 fertilizer experiments were carried out in different locations in the State of São Paulo, each of them with 27 treatments combinations arranged in three blocks of nine plots each, confounding two degrees of freedom from the NxPxK interaction (this design was defined as "group W" by F. Yates).

Doses of N (N) were 0, 60 and 120 kg/ha. For P_2O_5 (P) and K_2O

(K) the same amounts were applied: 0, 75 and 150 kg/ha.

Analysis of variance was performed in each experiment, resulting in the elimination of one of them from the final data set (on the account of a high residual mean square) in an atempt to avoid heteroscedasticity. In doing so, the ratio between the highest and the lowest residual mean squares drops down to 4.7. Therefore, the **specific residual technique** (COCHRAN & COX, 1957) was applied for better estimation of interesting effects.

Fertilizer prices per metric ton and the amount of nutrients (in percent) are given in **Table 1**.

Table 1. Fertilizer prices.

Fertilizer	Price	Nutrient (%)
Potassium chloride (t)	R$ 357.39	60 (K_2O)
Simple superphosphate (t)	R$ 227.80	20 (P_2O_5)
Urea (t)	R$ 444.48	45 (N)

Factor levels were coded for fitting the response surface:

N*=N/60, P*=P/75, K*=K/76.

Finally, all analyses were performed using the SAS System.

RESULTS

Analysis of Variance for the Group of Experiments

Table 2 gives the ANOVA results for the fourteen experiments according to the specific residual technique used (the specific residual was inserted after its specific effect).

The quadratic effects of N and P, and the N'x P' interaction were not significant. There are evidences of significant effects for N'xK' and P'xK' (at 6% level in the former and 9% in the latter). The polynomial surface obtained fitted very well to the data (R^2=0,90), and the lack of fit was not significant. The equation was obtained through the REG procedure:

$$Y = 103.77 + 4.57N + 4.30P + 18.79K - 5.61\,K^2 + 2.43\,NK - 1.07\,PK.$$

Table 2. Analysis of variance.

Source of Variation	d.f.	Sums of Squares	Mean Squares	F	Prob.
EXPERIMENTS (E)	13	157,587.27	12,122.10	16.460	0.0001**
BLOCKS(E)	28	10,603.32	378.69		
N'	1	12,363.40	12,363.40	26,500	0.0002**
ExN'	13	6,064.79	466.52		
N''	1	271.56	271.53	0.871	0.3677ns
ExN''	13	4053.56	311.81		
K'	1	20,153.62	20,153.62	55.020	0.0001**
ExK'	13	4,762.07	366.31		
K''	1	2,642.46	2,642.46	22.760	0.0004**
ExK''	13	1,509.4	116.11		
P'	1	2,630.64	2,630.64	9.710	0.0076**
ExP'	13	3,522.29	270.94		
P''	1	45.27	45.27	0.490	0.4960ns
ExP'	13	1,199.67	92.28		
N'xK'	1	996.21	996.21	4.440	0.0550ns
ExN'xK'	13	2,913.26	224.10		
N'xP'	1	0.97	0.97	0.0004	0.9843ns
ExN'xP'	13	3,073.36	236.41		
P'xK'	1	191.78	191.78	3.510	0.0820ns
ExP'xK'	13	709.35	54.56		
NxPxK	6	2,849.34	474.89	3.210	0.0046**
Error	264	39,000.88	147.73		
Corrected Total	377	268,500.10			

A lack of fit test was performed; results are given in **Table 3.**

Since there were no quadratic effects for N and P, no optimum level was reached for the response variable with regard to these two nutrients.

Table 3. Lack of fit test.

Source of Variation	d.f.	Sums of Squares	Mean Squares	F	Prob.
EXPERIMENTS (E)	13	157,587.27	12,122.098	16.46	0.0001**
BLOCKS (E)	28	10,603.32	378.690		
REGRESSION	6	38,978.13	6,663.02	36.47	0.0001**
LACK OF FIT	18	4,335.32	240.85	1.32	0.173ns
PURE ERROR	312	56,996.06	182.68		
CORRECTED TOTAL	377	268,500.10			

Per Unit Cost Production Function

Accordingly, the G function is defined as:

$$G(N,P,K)=\frac{103.77+4.57N+4.23P+18.79K-5.61K^2+2.45NK-1.07PK}{550+59.26N+85.42P+44.67K};$$

27 points were calculated for this function. An iterative process was started by fitting a second order polynomial to these points. The equation obtained was:

$$H_1(N,P,K) = 0.1883 - 0.0095N - 0.0199P + 0.0149K + 0.0018P^2 -$$

$$- 0.0078K^2 + 0.0017NP + 0.0034NK - 0.0007PK.$$

The series of contour plots shown in **Figure 1** were constructed by fixing each of the three levels for the three factors. The gray scale varies from white (highest value) to black (lowest one). It can be viewed that the highest values are associated with level zero of N and P.

When searching the optimum response, the first partial derivatives were calculated, leading to the following equations:

$$\frac{\partial H_1}{\partial H} = -0.0095 + 0.0017P + 0.0034K = 0,$$

$$\frac{\partial H_1}{\partial P} = -0.0199 + 0.0017N + 0.0036P - 0.0007K = 0,$$

$$\frac{\partial H_1}{\partial K} = -0.0149 + 0.0034N - 0.0007P - 0.0156K = 0.$$

The solution of this system corresponds to the point

P(N = - 30.778; P = - 18.779; K = 6.595).

The Hessian matrix is:

$$\begin{bmatrix} 0 & 0.0017 & 0.0034 \\ 0.0017 & 0.0036 & -0.0007 \\ 0.0034 & -0.0007 & -0.0156 \end{bmatrix}.$$

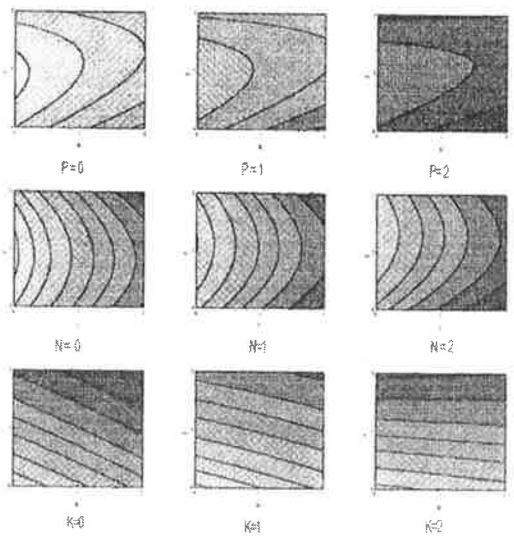

Figure 1: Contour plots for G(N,P,K)

According to its main diagonal values, it shows that the fitted surface does not have a point of maximum, minimum or a col. Therefore, the absolute maximum has to be determined on the borderline of the experimental region.

Taking N = P = 0, according to the contour plots of **Figure 1**, the reduced function is:

$$H_1 = 0.189 + 0.0141K - 0.0078K^2,$$

which has a maximum at K = 0.90. Consequently, the absolute maximum was found at P(N = 0; P = 0; K = 0.90).

A new iteration was performed, based on 27 points calculated for the G function, this time in the neighbourhood of the approximate optimum already found. In this case, N and P values were 0.0, 0.05 and 0.10, and the values for K were 0.5, 1.0 and 1.5. **Figure 2** shows schematically the strategy described.

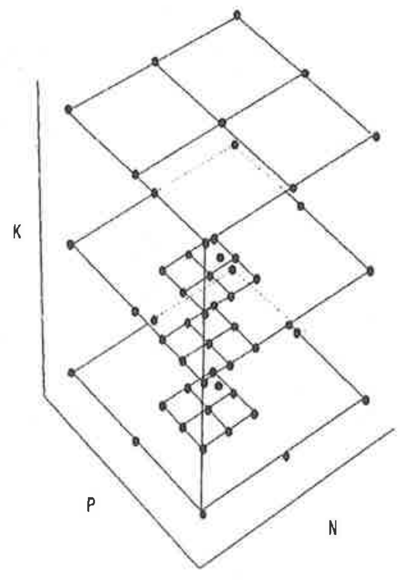

Figure 2: First and second iterations

The second order polynomial equation fitted to that grid is:

$$H_2(N,P,K)=0.1897 - 0.0127N - 0.0233P+0.0159K+0.0007N^2+0.0031P^2 -$$

$$- 0.0090K^2+0.0032NP+0.0049NK+0.0004PK.$$

Again, taking N = P = 0, the maximum was found at P(N = 0; P = 0; K = 0.88).

A third iteration yields P(N = 0; P = 0; K = 0.89) and, in view of these facts, the maximum is considered reached at

P(N = 0; P = 0; K = 0.89).

The value of the G function calculated at P(N = 0; P = 0; K = 0.89) is 0.197 t/real and, from **Table 4**, it can be easily inferred that this process successfully reached the absolute maximum.

Table 4. G values for N, P_2O_5 and K_2O.

N	P	K	G	N	P	K	G	N	P	K	G
0	0	0	0.1887	1	0	0	0.1778	2	0	0	0.1689
0	0	1	0.1967	1	0	1	0.1895	2	0	1	0.1836
0	0	2	0.1860	1	0	2	0.1837	2	0	2	0.1818
0	1	0	0.1701	1	1	0	0.1621	2	1	0	0.1555
0	1	1	0.1767	1	1	1	0.1720	2	1	1	0.1680
0	1	2	0.1670	1	1	2	0.1664	2	1	2	0.1659
0	2	0	0.1559	1	2	0	0.1499	2	2	0	0.1448
0	2	1	0.1612	1	2	1	0.1581	2	2	1	0.1554
0	2	2	0.1521	1	2	2	0.1526	2	2	2	0.1530

CONCLUSION

- It seems that the methodology developed in this paper solves satisfactorily the problem of maximizing the G function.

- Second order polynomials aproximated well the G function.
- The process seems to converge fast.

An important fact that is worth warning is that the RSREG procedure (a component of the SAS system) was developed only for fitting **complete** second order polynomials; when the fitted surface corresponds to an "incomplete" one (like the cylindric paraboloid fitted in the example), this procedure yields incorrect results. Therefore, the equation parameters should be estimated using the REG or the GLM procedures; canonical analysis has to be performed with more specific software, or by hand, for it is a simple task in such polynomials.

SUMMARY

The aim of this work was to determine the nutrient doses that minimize the product cost in fertilizer experiments. In doing so, a function G was developed, with three explanatory variables. This function is defined as the ratio between a second order polynomial, fitted to crop productivity taken from experimental data, and a linear function of associated costs. A new second order polynomial was fitted iteratively, in order to approach the behavior of the G function, and the corresponding canonical analysis was performed to determine its maximum point. This optimum yields the maximum amount of product obtained by monetary unit, and, analogously, the minimum point of G^{-1} yields the minimum cost by unit produced. The G function was very well approached by second order polynomial; the methodology presented dealt well with the problem and converged rapidly. Some remarks are presented with regard to the use of the SAS software as a tool to deal with such a problem.

Key words: Minimization of product cost, experiments with fertilizers, iterative method of solution of second degree system of equations.

RESUMO

A MINIMIZAÇÃO DO CUSTO DO PRODUTO AGRÍCOLA EM EXPERIMENTOS COM FERTILIZANTES

Esta pesquisa teve por fim determinar doses de nutrientes que minimizem o custo do produto em experimentos com fertilizantes. Para isso, foi determinada uma função G, a três variáveis, obtida pelo quociente de um polinômio de segundo grau ajustado à produtividade estimada a partir de dados experimentais, e uma função linear do custo fixo e dos custos dos nutrientes. Esta função G é, na verdade, a produtividade obtida por unidade monetária (real ou dólar) e, pois, a função G^{-1} corresponde ao custo por unidade do produto.

A função G(N,P,K), no caso mais geral, é válida apenas para valores não-negativos de N, P e K, e não pode ter assíntotas verticais, pois o denominador só se pode anular para pontos fora de seu domínio. A função que realmente interessa é G^{-1}, que dá o custo por unidade monetária. Mas G^{-1} pode tender a infinito dentro do seu domínio de definição, o que complica seu estudo. Daí a preferência pela função G, cujo ponto de máximo corresponde exatamente ao ponto de mínimo G^{-1}, que se quer obter.

O método usual para calcular o ponto de máximo de G seria a resolução do sistema

$$\partial G / \partial N = \partial G / \partial P = \partial G / \partial K = 0,$$

mas este sistema, de três equações de segundo grau a três incógnitas, é numericamente de resolução muito complexa na prática (REY PASTOR et al., 1969). O método usado, mais conveniente, foi a aproximação da função G(N, P, K) por um polinômio de segundo grau, em iterações sucessivas, que conduzem rapidamente à solução desejada. Algumas restrições ao programa RSREG do SAS, para utilização deste método, são apresentadas.

Palavras-chave: Minimização do custo do produto, experimentos de adubação, método iterativo de resolução de sistema de equações de segundo grau com várias incógnitas.

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